

GRAB BAG

- Measures of central tendency
- Analyzing data

Getting Ready

What You'll Need

Cuisenaire Rods, 1 set per pair

Paper bags, 1 per pair

Activity Master, page 104

Overview

Students grab a handful of Cuisenaire Rods and find the mean, median, and mode of the lengths of those rods. Then they modify the contents of the bag and predict the mean, median, and mode of the lengths of the new set of rods. In this activity, students have the opportunity to:

- investigate mean, median, and mode
- draw conclusions about a data set
- make and test hypotheses

Other *Super Source* activities that explore these and related concepts are:

Geo-Hoops, page 51

How High? How Long?, page 56

Rocket Launch, page 60

The Activity

On Their Own (Part 1)

Hector insisted that anytime he picked rods out of the class storage box, there would always be a blue rod in his pick. He therefore thought that blue must be the average length for a rod. What do you think? Suppose you reach into a bag and grab a handful of Cuisenaire Rods. What is the typical length of a rod in your handful?

- Each color of Cuisenaire Rod is a different length. The white rod is the shortest at 1 centimeter long; the orange rod is the longest at 10 centimeters long. Note: You may find it helpful to make a chart of rod colors and their corresponding lengths.
- Work with a partner. Put one set of Cuisenaire Rods in a paper bag.
- Player 1 should grab a handful of rods and place them on the desktop, recording the number of each color of rod in the handful.
- Find and record the median length of the handful. Here's how:
 - ◆ Organize the rods from longest to shortest. Find the middle rod. If you have an even number of rods, there will be two rods in the middle. If the middle rods are different colors, the median is halfway between their lengths. If the two rods are the same color, the length of that color is the median.

- Find and record the mode length of the handful. Here's how:
 - ◆ Sort the handful into piles by color. The pile with the most rods is the color that represents the mode. You can have more than one mode.
- Find and record the mean length of the handful. Here's how:
 - ◆ Find the sum of the lengths in the handful. Divide the sum by the number of rods in the handful. Round to the nearest tenth.
- Put the rods back into the bag and have Player 2 grab a handful. Find the mean, median, and mode of the lengths in the handful.
- Now combine the data from both handfuls of rods. Add the rods you grabbed to those in your partner's handful and find the combined mean, median, and mode.
- Be ready to explain what the measures tell you about the typical lengths of rods in a handful.

Thinking and Sharing

Invite pairs to share their work. Create a chart to display each pair's results.

Use prompts like these to promote class discussion:

- Which measure (median, mode, or mean) best represents the typical length of a rod from a handful of Cuisenaire Rods? Explain your reasoning.
- What does the mode tell you about the handful? Why is the mode sometimes very different from the mean and median?
- What does the median tell you about the handful of rods?
- How is the mean useful in describing the typical length of a rod?
- Can you use the mean from both trials to find the combined mean? Explain.
- Which set of data (Player 1, Player 2, or the combined trials) is most useful in describing the typical length of a rod? Why?
- Do you notice any patterns occurring in the class data as a whole?

On Their Own (Part 2)

What if... the contents of the bag were changed? Suppose two orange, two blue, and two brown Cuisenaire Rods were removed from the bag. How would this affect the typical length of a rod in a handful of Cuisenaire Rods?

- First predict the results of your experiment. Predict the median, mode, and mean of a handful of Cuisenaire Rods. Be ready to explain your predictions.
- Take turns grabbing a handful of Cuisenaire Rods from the bag.
- For each handful, find the median, mode, and mean of the lengths of the rods.
- Now combine the data from both handfuls and find the median, mode, and mean of the lengths of the combined rods.
- Be ready to explain the results of your experiment and prediction.

Thinking and Sharing

Have pairs share their predictions and the results of their combined trials. Compile the predictions on a class chart. Use prompts like these to promote class discussion:

- How did you go about making your predictions?
- How did the data you collected compare to your predictions?
- Was it easier to predict the median, mode, or mean? Why?
- What do you notice about the class's predictions? How are they the same? How do they differ?
- Were you surprised by any of the results of your experiment? Explain.
- Compare your data to that gathered during the first activity. Which measure—the mean, median, or mode—was affected most by the removal of the rods? Why?



The sports writer of a newspaper wrote that a baseball pitcher typically throws 92 miles per hour. What does this statistic really mean? What information would help you to understand the statistic? Write a letter to the sports writer explaining what kind of information readers need to know in order to understand this statistic.

Teacher Talk

Where's the Mathematics?

Students investigate the meaning of the three measures of central tendency: mean, median, and mode. As they analyze the lengths in a handful of Cuisenaire Rods, they explore what kinds of information are needed to make statistics meaningful.

Each pair of students begins by putting a full set of Cuisenaire Rods in a paper bag. The set should include 4 orange, 4 blue, 4 brown, 4 black, 4 dark green, 4 yellow, 6 purple, 10 light green, 12 red, and 22 white rods. The results of the activity will not be affected greatly if two or three smaller rods are missing from the set.

The following set of rods may be similar to a handful taken by your students.

Rods in Handful	Total length by color	Mode, Median and Mean of Handful
2 orange	20 cm	Mode = 9 cm
4 blue	36 cm	
3 brown	24 cm	Median = 8 cm
2 dark green	12 cm	
2 yellow	10 cm	
1 purple	4 cm	Mean = 7.3 cm
1 light green	3 cm	
15 total rods	109 total cm	

The following data sets may be similar to those collected by your students in the first activity.

	Student A	Student B	Combined	Student A	Student B	Combined
Median (cm)	6	6.5	6	7	7	7
Mode (cm)	8, 1	9	8	9	9	9
Mean (cm)	5.6	6.2	5.8	6.9	6.7	6.8
Number of Rods	19	12	31	19	18	37
Total Length (cm)	107	74	181	132	121	253

Students may associate the word *average* with the mean. Actually, all three measures of central tendency can be called an *average* or a *measure of the center* of a set of data. Each measure of central tendency helps you understand how the data in a set are distributed.

As students investigate the median, you may hear statements like, “The median is the middle length so about half of the rods in the handful are bigger and half are smaller” and “The median tells you the middle but it doesn’t tell the biggest and smallest pieces in the handful.” It is possible that all the rods in the handful are the same length. Without additional knowledge, the median gives a narrow perspective of the data.

Students may be surprised to find that the mode often varies widely from the mean and the median. They may also have difficulty understanding in what situations the mode becomes a useful piece of information. Two real-life examples are T-shirt sizes or tire sizes; students may think of other examples. When the same mode occurs in several handfuls, students may use the information to ask, “Why do we tend to grab more of the longer rods than the shorter ones?” and “Why is the mode usually a longer length of rod even though there are many more short rods in the bag than long ones?”

Students are accustomed to finding the mean and usually refer to it as the average. This custom is followed widely by newspapers and even textbooks, although both the median and the mode are also measures of average. Students learn early on how to add up all the elements and divide by the number of elements, but they often have trouble understanding how the mean represents the set of data. Students may observe that as the median is the middle rod, the mean is the middle of the numbers themselves.

Arguments can be made for any of the measures of central tendency to best represent the average length in a handful of Cuisenaire Rods. As students explore the ideas, they may reach the conclusion that it is most helpful to have more than one of the measures in order to understand the data. They may also conclude that it is helpful to know the range and the number of rods in the handful.

Students may be surprised to find that you can’t average averages. To help students understand why it doesn’t work, have them discuss the following situation: A baseball player gets only 5 at bats in the first half of the year but gets 4 hits for an .800 batting average. During the second half of the year the player gets 80 hits out of 250 at bats for a .320 average. Averaging the averages gives the player a .560 batting average for the year, but does this average really represent his hitting ability? His actual batting average for the year would be .329 ($84 \text{ hits} \div 255 \text{ at bats} = .329$).

The combined trials should yield the most useful information in describing the typical rod length in a handful of Cuisenaire Rods simply because the analysis is based on a larger pool of data. Some students may be concerned that the measures are not accurate because the size of students’ hands varies, and so does the number of rods they can grab. Actually, this is one of the reasons why the mean, median, and mode are useful measures: They allow statisticians to compare samples that are not equal in size. Students may suggest compiling data from other groups. If students try this, they may find that there is a point at which new data actually adds nothing new to their understanding of the situation. Likewise, statisticians limit the number and size of the samples they gather.

In the second activity, students will probably predict a lower mean, median, and mode than they found in the first activity. With fewer long rods in the bag, students may find that the mode is very difficult to predict and it may be much lower than the median and mean.

The following data sets may be similar to those collected by your students in the second activity.

	Student A	Student B	Combined	Student A	Student B	Combined
Median (cm)	4	3	3.5	6	6.5	6.3
Mode (cm)	4, 2	1	4, 2, 1	3	7	7, 3
Mean (cm)	4.0	2.7	3.4	5.7	6.3	7.2
Number of Rods	12	11	23	14	14	28
Total Length (cm)	48	30	78	80	121	201